

Code: 23BS1201

I B.Tech - II Semester – Regular Examinations - JULY 2024**DIFFERENTIAL EQUATIONS & VECTOR CALCULUS**
(Common for ALL BRANCHES)

Duration: 3 hours

Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.

2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.

3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.

4. All parts of Question paper must be answered in one place.

BL – Blooms Level

CO – Course Outcome

PART – A

		BL	CO
1.a)	Check whether the equation $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$ is exact differential equation or not.	L2	CO1
1.b)	Find the integrating factor of $x \frac{dy}{dx} + y = \log x$.	L2	CO1
1.c)	Define Auxiliary equation and Wronskian.	L1	CO2
1.d)	Find the Particular integral of $(D^3 + 4D)y = \sin 2x$.	L1	CO2
1.e)	Form a partial differential equation by eliminating arbitrary constant 'a' from $Z = a \log \left(\frac{b(y-1)}{1-x} \right)$.	L3	CO2
1.f)	Form a partial differential equation by eliminating arbitrary function 'φ' from $Z = e^{my} \varphi(x - y)$.	L3	CO2
1.g)	Define directional derivative and gradient of a scalar point function.	L1	CO3

1.h)	Find the $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.	L3	CO3
1.i)	State the Green's theorem.	L2	CO5
1.j)	If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, Calculate, $\int_C \vec{F} \cdot d\vec{r}$ where 'C' is the curve in the xy-plane $y=2x^2$ from (0,0) to (1,2).	L3	CO5

PART – B

			BL	CO	Max. Marks
UNIT-I					
2	a)	Solve the differential equation $xy(1 + xy^2) \frac{dy}{dx} = 1$.	L3	CO2	5 M
	b)	Find the solution to the differential equation. $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$	L1	CO4	5 M
OR					
3	a)	A bacterial culture, growing exponentially increases from 200 to 500 grams in the period from 6 am to 9 am. How many grams will be present at noon?	L2	CO4	5 M
	b)	Calculate the general solution of the differential equation $(y \log y)dx + (x - \log y)dy = 0$.	L3	CO4	5 M

UNIT-II

4	a)	A condenser of capacity C discharged through an inductance 'L' & resistance R in series and the charge 'q' at time 't' satisfies the equation $L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{c} = 0$, given that L=0.25 henries, R=250 ohms and $C = 2 \times 10^{-6}$ farads, and that when t=0, charge q is 0.002 coulombs and the current $\frac{dq}{dt} = 0$, obtain the value of 'q' in terms of 't'.	L4	CO2	5 M
	b)	Solve $(D^2 + D + 1)y = (1 - e^x)^2$.	L3	CO2	5 M

OR

5	a)	Calculate the particular integral of $(D^3 + 1)y = \sin(2x + 3)$.	L3	CO2	5 M
	b)	Solve $(D^2 + 4)y = \tan 2x$.	L3	CO2	5 M

UNIT-III

6	a)	Solve the partial differential equation $(D^2 - 2DD' + D'^2)z = e^{x+y}$.	L3	CO2	5 M
	b)	Determine the solution to the equation $xp - yq = y^2 - x^2$.	L3	CO4	5 M

OR

7	a)	Solve $\frac{y^2z}{x}p + xzq = y^2$.	L3	CO2	5 M
	b)	Form a partial differential equation by eliminating the arbitrary constants from the differential equation of all spheres whose centres lie on the z-axis.	L2	CO2	5 M

UNIT-IV					
8	a)	Find the directional derivative of $\phi = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the normal to the surface $x \log z - y^2 = -4$ at (-1,2,1)	L4	CO5	5 M
	b)	Prove that $\text{div}(r^n \vec{r}) = (n + 3)r^n$	L3	CO3	5 M
OR					
9	a)	Show that $\nabla^2(r^m) = m(m + 1)r^{m-2}$	L3	CO3	5 M
	b)	Identify the values of 'a' and 'b' such that the surface $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at (1,-1,2).	L2	CO3	5 M
UNIT-V					
10	Verify Green's theorem $\int_C (3x - 8y^2)dx + (4y - 6xy)dy$ where 'C' is the boundary of the region bounded by the $x=0$, $y=0$ and $x+y=1$.		L4	CO5	10 M
OR					
11	Calculate $\int_S \vec{F} \cdot \vec{n} ds$ where $\vec{F} = 2x^2y\vec{i} - y^2\vec{j} + 4xz^2\vec{k}$ and 'S' is the closed surface of the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$, and the planes $x=0$, $x=2$, $y=0$ and $z=0$.		L3	CO5	10 M